Optimizing Precision of Self-Localization in the Simulated Robotics Soccer

Vadim Kyrylov[1], David Brokenshire[1], Eddie Hou[1]

[1] Simon Fraser University – Surrey
Surrey, British Columbia V3T 2W1 Canada
{vkyrylov, brokenshire, ehou}@sfu.ca
http://www.surrey.sfu.ca/

Abstract. We show that all published visual data processing methods for the simulated robotic soccer so far were not utilizing all available information, because they were mainly based on heuristic considerations. On the other hand, we show that the improved accuracy could result in a significant score gain even for a reasonably good simulated team. Some researchers have approached to estimating the agent location and orientation as two separate tasks. This had caused systematic errors in the angular measurements. Further attempts to get rid of them (e.g. by completely neglecting the angular data) only aggravated the problem and resulted in the losses in he accuracy. We treat the estimation problem in a rigorous way, by jointly estimating the agent Cartesian coordinates and its view direction angle using the extended Kalman filtering technique. The experiments with the proposed method run in the simulated soccer setting give the idea of the achievable average error limit for this particular application and provide necessary guidance for the implementation. This limit proved to be about 25-33 per cent lower than that of the best algorithms published by far.

1 Introduction

This paper is the result of our development of the SFUNLEASHED'03 team for the simulated robotic soccer competitions. The work was started in the fall 2001. While trying to improve the BASIC UVA team [2, 5] used as a prototype, we realized that better, or maybe best possible, utilization of the information provided by the visual sensor was an important task. This is especially critical for self-localization and performing low-level actions such as scoring and ball interception by the players. However, in this paper we do not consider processing of measurements made over time. Rather, we are dealing with a snapshot of visual sensor data and are leaving the study of the data filtering dynamics to our future publications.

By far, processing visual sensor information with the purpose of self-localization and object tracking has been addressed in different ways by several researchers in the simulated robotic soccer community. The related works could be split into two groups, elaborating either on the deterministic or stochastic approaches.

1.1 Deterministic methods

In fact, the visual sensor in the simulated soccer always supplies deterministic data. This feature has recently given birth to a method based on two-dimensional interval arithmetic, implemented in the LUCKY LÜBECK simulated soccer team [4]. However, methods based on interval arithmetic have a general shortcoming: they all are risk-aversive. This results in that the obtained interval estimate could have an exaggerated size of the area to which the observed object is believed to belong. It is also unknown if this method provides the best possible accuracy, since there is no explicitly stated optimality criteria. Another shortcoming is the lack of attempts to eliminate the systematic errors. Indeed, in the simulated soccer, while the quantization distortions are different for each individual object, the neck direction errors are making biased the visual sensor readings of all visible objects. However, in [4] these indeed different errors are being treated without elaborating on their differences, which inevitably leads to a loss of some information.

One more recent work done at the Humboldt University of Berlin, uses the optimization of the mismatch between the readings of several landmarks using a gradient descent function [1]. This method offers a mathematically elegant solution, which has resulted in significantly higher precision than some simple heuristic algorithms. However, the weak point is that the angular measurements are being completely ignored in this implementation, which means that all the potential of the simulated visual sensor has not been completely utilized. We speculate that angular measurements were omitted due to the presence of systematic errors in them.

1.2 Stochastic methods

They have the right to exist because the complicated player movements, taken together with the several quantization transformations implemented in the Soccer Server [3], could be regarded a good generator of pseudo random sequences. Subtle notions of using stochastic estimation methods are spread over all the RoboCup literature. However, very few, if any, comprehensive overviews have been published. One early exception is the detailed description of the visual sensor data processing algorithm in [6]. For self-localization, the agent neck direction angle is estimated first. Then a weighted sum of the individual estimates of the agent position relative to different flags is used. The weights are inversely proportional to the distances from the agent to these landmarks. This heuristic approach appears to work fairly well, but it neglects the systematic error caused by the imprecision of the player neck angle. Therefore, this is not the best possible method.

A rather thorough investigation of several methods can be found in [2]. In this work, the so-called particle filter has been selected for visual sensor data processing over two simple heuristic methods and the classic Kalman filter. This method was implemented in the BASIC UVA soccer team [5], which we are using as a prototype. Compared to the nearest-flag method for agent self-localization, the particle filter reportedly yielded almost three times better accuracy. The downside was the two order of magnitude increase in execution time required to obtain this gain. With this reservation, it looks like using the particle filter for performing the agent self-localization task is overkill, and the grounds on which less resource-demanding Kalman filter had been rejected are not all convincing. Another reservation, however, is that the agent neck direction angle was estimated separately from its Cartesian coordinates, which caused systematic errors in angular measurements of the landmarks. This is a plausible explanation of that the authors of this thesis elected to neglect angular information in favor of the range measurements.

This allows us to conclude that not all the potential gain has been obtained from the application of stochastic methods to visual sensor data processing in the simulated soccer. It is still unclear, where the limit of the perfection lies and how much could be in principle obtained by improving the data processing algorithms.

The purpose of this paper is two-fold. Firstly, we want to synthesize near-optimal algorithms for estimating object locations by a soccer agent using multiple readings received in one cycle. Secondly, we want to find the limits of perfection for visual sensor data processing methods in the simulated robotic soccer.

We are pursuing these goals by a comprehensive stochastic analysis of the visual sensor of the simulated soccer player, taking into account the previously neglected factors, such as Cartesian coordinate measurement correlation and the presence of the systematic errors in the angular measurements. Here we consider the static case only and deliberately do not use filtering data over time, leaving the multi-cycle case for the future. Because for the estimation algorithm synthesis we are using the extended Kalman filtering technique, it can be generalized for this case rather easily.

The paper is organized, as follows. In Section 2 we present the results of our study of potential benefits from gaining higher accuracy of the simulated visual sensor. Section 3 explains how the visual sensor works from the stochastic standpoint. In Section 4 we formulate the self-localization optimization problem and outline its rigorous solution based on the extended Kalman filtering technique. Section 5 describes the simulation results, and Section 6 concludes this study.

2 Expected Benefits

Prior to committing the development of an improved simulated soccer team, it would be prudent to try to understand the potential benefits of enhanced visual data processing and compare them with the anticipated costs. While the costs could be estimated in a straightforward way, because they include the development effort, which we can measure easily, measuring the benefits appears to be more challenging. One of the reasons is that for different teams the improved precision of the coordinate information would likely result in different gains. While the strengths of some teams could critically depend on the quality of visual data, for some other teams this factor could be less significant, especially if they are more relying on the elaborated high-level player skills and team tactics.

With this reservation in mind, we have conducted a set of simulations with only one simulated team, the BASIC UvA. The major reasons for this choice were the availability of a well-understood source code [5] and supporting documentation [2].

The objective of this study was to estimate the influence of raw visual data quality on two high-level performance criteria, such as the win/lose ratio and the score. The experimental results are summarized in the Table 1.

Table 1. Team Performance with Different Visual Sensor Precision

Error multiplier	**Game statistics**				**Score statistics**		
	Wins	**Losses**	**Draws**	**Win/Loss**	**Difference**	**Mean**	**St.dev.**
×0.0	42	24	35	1.75	96-69=27	0.27	1.14
×0.5	39	23	39	1.70	73-55=18	0.18	1.11
×1.0	38	30	33	1.27	89-77=12	0.12	1.26
×1.5	28	48	25	0.58	76-116=-40	-0.40	1.52
×2.0	12	61	28	0.20	57-149=-92	-0.92	1.42

For running the experiments, the Soccer Server was modified so that it supplied the default visual sensor data to one team, while the second team was receiving the data whose precision was under our control. Both teams were modified so that the visual data reported to agents were float-point coordinate values instead of integers. In all other aspects, the teams were identical to the BASIC UvA.

Each experiment included a set of 101 games with the error multiplier for one team selected from the set {0, 0.5, 1.0, 1.5, 2.0}. In other words, the visual sensor errors for this team ranged from zero to double the default value.

It can be seen how the performance criteria are responsive to the visual sensor error. Because increased error leads to a significant loss, BASIC UVA has indeed a fairly good visual sensor processing algorithms. By improving the precision two times, we can expect about 1.5 times higher chance of winning the game, or increasing the score difference by about 50%. However, further reduction of the error to zero would result in almost insignificant gain. This gives a clue on where the limit of perfection lies for this simulated team.

The unexpected asymmetry of the results in the case, when the errors are equal for both teams, can be explained by high volatility of the score difference, and especially of the win/loss ratio. With this volatility, the bias proved to be statistically insignificant even at 0.01 confidence level. So, we believe that the results give the idea of how much could be gained from reducing errors in the visual sensor data.

3 Visual Sensor Raw Data in the Simulated Soccer

In simulated soccer the visual sensor returns measurements of the polar coordinates of objects, i.e. the range and the direction, with regular time intervals. The range to the object is measured relative to current agent location in the field, and the direction is measured relative to the agent neck orientation. The precise data are distorted by rounding errors [3]. They could be regarded random, assuming that the locations of the visible objects or of the agent itself are the source of this randomness.

Landmarks that can be used for self-localization are borderlines and flags. Using a borderline, the agent can only determine the angle between the line and the direction it is facing. For the rest objects both direction and range measurements are available. In Section 4 below, we will show that the borderlines could be treated as special case of flags.

Quantization of the angular measurements results in the random error uniformly distributed within ±0.5°. The precision of the simulated visual sensor readings is significantly different for the range and the direction measurements. With current Soccer Server settings, the magnitude of the range error measurement in a given simulation cycle is proportional to the actual range and is uniformly distributed in the ±5% interval for moving objects and the ±0.5% interval for landmarks, which are used for agent self-localization [3].

The direction measurement error, however, is absolute and is uniformly distributed within ±0.5°. Fig.1a gives the idea of the shape of the ambiguity area, assuming that the agent global neck position is known precisely. Because the range and direction errors are statistically independent by design, we can conclude that in the quasi-rectangle BCDE they have jointly uniform distribution.

Note that the localization error for flags caused by range and angular errors, respectively, has a 0.6/1.0 ratio (Fig.1a). Therefore, we speculate that complete negligence of the angular data would result in about one third loss of precision, if no systematic error is present. (We address the systematic error in Section 4 below.) As it can be seen from Fig.1a, for the case of objects, such as the ball and players, the ambiguity area has 6/1 elongation ratio. We anticipate that negligence of the angular data would result the loss of precision of one order of magnitude even in the presence of the systematic error. In particular, angular data appears to be especially useful for the improvement of ball tracking using inter-agent communication (Fig.1b). Therefore, we trust that mathematically correct treatment of angular data is crucial.

The elongated ambiguity area also translates into the high correlation of the Cartesian coordinate measurements supplied by the visual sensor in most scene configurations. This factor has been ignored in all published methods so far. In order to propose a rigorous solution to the coordinate data processing problem for the general case, this correlation must be taken into account, and its contribution properly assessed.

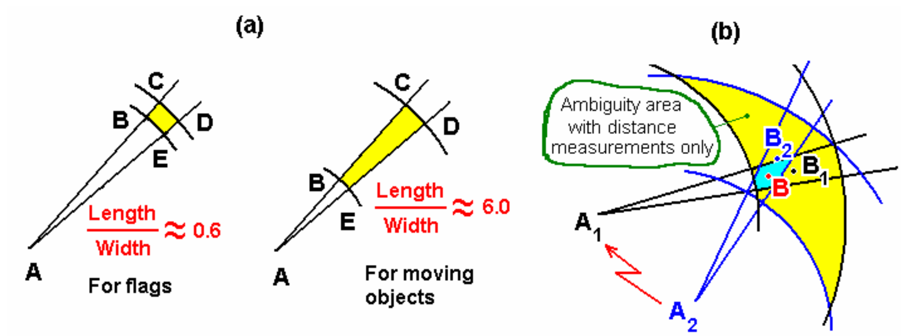

Fig.1. The ambiguity area for the visual sensor measurements *(a)* and ball localization using data from two agents *(b)*

In the simulated soccer, we have two standard problems based on visual sensor data utilization: agent self-localization and moving object tracking. The solutions to both problems are very similar and differ only in minor details. In what follows, we will be dealing with self-localization and pointing to the difference of object tracking only when necessary.

With their visual sensor, agents can only measure angles relative to their head position, i.e. the bisector of their field of view. The neck direction, used as the reference for the visual sensor, is known to the agent with some errors. Field borderlines have been normally used for estimating the agent neck direction by some authors so far [2,4,5]. In what follows, however, we are temporarily assuming that the neck direction is known precisely.

Fig.2a shows how agent **A** is viewing a reference point **F** (which could be a landmark in self-localization, or a ball, or another agent in object tracking). Assuming that the agent belongs to the left team, the positive direction of angles and absolute Cartesian coordinate axes, are as shown. The agent's line of sight can have any direction β. From the Soccer Server the agent receives values r and d, which are the estimated range and direction to **F**, respectively.

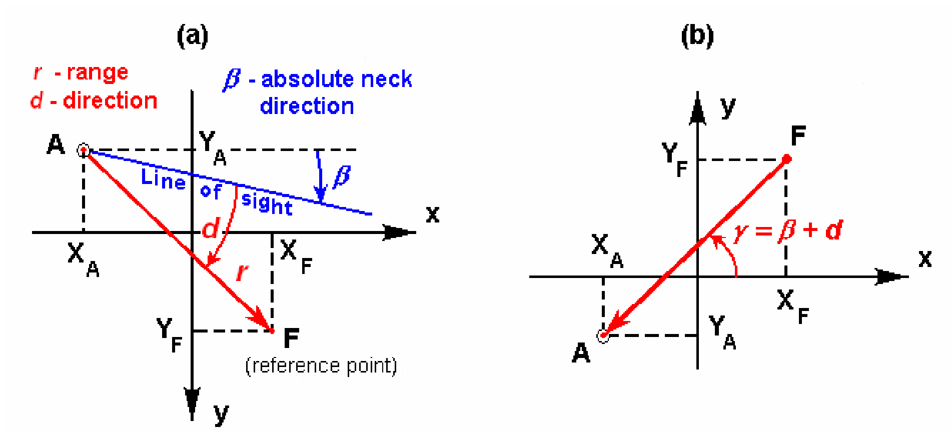

Fig.2. Agent viewing geometry *(a)* and preferred absolute coordinate system for agent self-localization *(b)*

Note that, given only one reference point, and the global neck direction, an agent can in principle estimate its location (hence the name 'one-flag method' [2]).

However, because the neck direction angle in existing self-localization methods is estimated with some error, a systematic error is present. This error biases all angular measurements returned by the agent visual sensor. Fig. 3 illustrates the situation, when the agent is viewing several flags and the random angular measurement errors are all assumed to be negligently small. Precise directions are shown as thin lines, while the thicker lines show the measurements distorted by the systematic error δ.

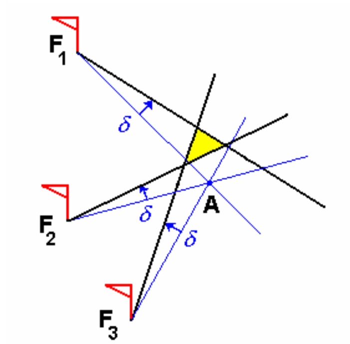

Fig.3. The perceived ambiguity area with the systematic error δ added to the angular measurements

With the self-localization algorithms proposed in [2,4,5], the estimated agent location would be somewhere close to the center of the shaded triangle, which even does not cover the true agent location **A**. Adding more flags would hardly improve the situation, because each measurement is equally distorted by δ. We believe that this was exactly the reason of why some authors were reluctant about using angular data in self-localization and were relying on the range measurements only in the simulated soccer.

This indicates that we need a better estimation algorithm which would be addressing the bias in a consistent way.

In particular, we want to make two improvements to the methods proposed by our predecessors. We will be taking into account the correlation between measured Cartesian coordinates and eliminating the systematic error. In other words, we want to utilize all the potential of the visual sensor.

4 Using the Extended Kalman Filtering Algorithm

The complete utilization of the information supplied by the visual sensor could be achieved by jointly estimating the neck direction angle with the agent Cartesian coordinates. This is a standard approach developed in the stochastic estimation theory some three decades ago, in particular, with the adoption of the Kalman filtering technique [7].

4.1 The General Case

Prior to deriving optimal estimation procedure for our particular case, below we remind the general theory of the extended Kalman filter.

The objective is yielding an optimal estimate of the state vector **x** based on a set of (possibly indirect) measurements. The estimation procedure is recursive. Assuming that the filter has been already applied (*k*-1) times, on the *k*-th step its input parameters are, the measurement vector $\hat{\mathbf{z}}_k$ and its error covariance matrix $\mathbf{R}_k$. Other inputs also are the optimal state vector estimate obtained on the previous step $\hat{\mathbf{x}}_{k-1}$ and its covariance matrix $\mathbf{P}_{k-1}$. The filter returns an optimal estimate $\hat{\mathbf{x}}_k$ of the state vector and its covariance matrix $\mathbf{P}_k$ on current *k*-th step.

The optimality criterion is the maximal likelihood of $\hat{\mathbf{x}}_k$ [8]. With rather general assumptions about the state vector change pattern and the nature of the measurement errors, it is guaranteed that the resulting estimate has the least mean square error.

In many applications, the index *k* is normally being referred to as the discrete time; however, this is not a must. In the particular application discussed in the next sub section, *k* we will be standing for the index of the source providing raw data at the same instant of time.

In the general case, it is assumed that the state vector is changing over the discrete time *k* as

$$\mathbf{x_k} = \mathbf{f}(\mathbf{x_{k-1}}, \mathbf{w_{k-1}}), \tag{1}$$

where **f** is the state transition function, and **w** is the white Gaussian random sequence having zero mean and covariance matrix $\mathbf{Q}_k$.

Taken altogether, (1) specifies how the state vector is evolving over time, with both deterministic and random components possibly present in this process.

It is also assumed that the measurement model has the following form:

$$\hat{\mathbf{z}}_\mathbf{k} = \mathbf{h}(\mathbf{x_k}, \mathbf{v_k}), \tag{2}$$

where **h** is the observation transformation function and **v** is a white Gaussian measurement noise having zero mean and covariance matrix $\mathbf{R}_k$

If any of the functions **f**, **h** is nonlinear, the solution to the filtering problem is provided by the so-called *extended* version of the Kalman filter, which we are describing below.

The Kalman filtering algorithm is executed in three steps, as follows:

First, the *a priori* estimate $\tilde{\mathbf{x}}_k$ of the state variable vector and its covariance matrix $\tilde{\mathbf{P}}_k$ are computed using the model (1):

$$\tilde{\mathbf{x}}_\mathbf{k} = \mathbf{f}(\hat{\mathbf{x}}_\mathbf{k-1}, 0), \tag{3}$$

$$\tilde{\mathbf{P}}_\mathbf{k} = \mathbf{A_k P_{k-1} A_k^T} + \mathbf{W_k Q_{k-1} W_k^T}, \tag{4}$$

where $\mathbf{A}_k$, and $\mathbf{W}_k$ are the Jacobian matrices [8]:

$$A_{[i,j]} = \frac{\partial f_{[i]}}{\partial x_{[j]}}(\hat{x}_{k-1}, 0), \quad W_{[i,j]} = \frac{\partial f_{[i]}}{\partial w_{[j]}}(\hat{x}_{k-1}, 0) \cdot$$

Second, the Kalman matrix 'weight' is calculated:

$$\mathbf{K_k} = \tilde{\mathbf{P}}_\mathbf{k} \mathbf{H_k^T} (\mathbf{H_k} \tilde{\mathbf{P}}_\mathbf{k} \mathbf{H_k^T} + \mathbf{V_k R_k V_k^T})^{-1}, \tag{5}$$

where $\mathbf{H}_k$, and $\mathbf{V}_k$ are the Jacobian matrices:

$$H_{[i,j]} = \frac{\partial h_{[i]}}{\partial x_{[j]}}(\tilde{x}_k, 0), \quad V_{[i,j]} = \frac{\partial h_{[i]}}{\partial w_{[j]}}(\tilde{x}_k, 0) \cdot$$

Third, the optimal state variable vector estimate and its covariance matrix are computed:

$$\hat{\mathbf{x}}_\mathbf{k} = \tilde{\mathbf{x}}_\mathbf{k} + \mathbf{K_k}(\hat{\mathbf{z}}_\mathbf{k} - \mathbf{h}(\tilde{\mathbf{x}}_\mathbf{k}, \mathbf{0})), \tag{6}$$

$$\mathbf{P_k} = (\mathbf{I} - \mathbf{K_k H_k}) \tilde{\mathbf{P}}_\mathbf{k}. \tag{7}$$

It follows from (6) that the optimal estimate is a weighed sum of the a priori estimate $\tilde{\mathbf{x}}_k$ based on previously made observations and the innovation $(\hat{\mathbf{z}}_\mathbf{k} - \mathbf{h}(\tilde{\mathbf{x}}_\mathbf{k}, \mathbf{0}))$ containing the recent measurement. The matrix weight $\mathbf{K}_k$ is determined by the balance of precision of these two components. The more precision of the innovation, the greater elements of this matrix are. Therefore, if $\tilde{\mathbf{x}}_k$ contains significant errors, the contribution of the new measurements is greater. We will revisit this finding in Section 5 below.

4.2 Estimating the Agent State Vector in the Simulated Soccer

Now we customize the general Kalman filtering algorithm to the particular case of estimating the simulated soccer agent state vector.

We begin from the **state vector variable**. For the purpose of self-localization in this application, it contains two agent Cartesian coordinates, x, y and the neck direction angle, β:

$$\mathbf{x} = [x \quad y \quad \beta]^T \ . \tag{8}$$

Now consider the **model of the state vector dynamics** (1). The first thing that comes to mind is trying to model simulated soccer agent movement over time and applying the Kalman filtering technique. However, we speculate that this would not be making much sense (see the discussion in the next sub section). Rather, we preferred solving the self-localization problem for the soccer agent separately in each simulation cycle. (The only exception is that we are relying on the agent position obtained in the previous simulation cycle when less than two flags are visible; in this very rare case a heuristic algorithm is being used.)

So, from now on, we will be treating k in (1)-(7) as the landmark index in the list of landmarks visible in current simulation cycle. This allows us to reduce (3) to the trivial linear model

$$\mathbf{x_k} = \mathbf{x_{k-1}} \, . \tag{3a}$$

Moreover, the Jacobian matrices $\mathbf{A}_k$, and $\mathbf{W}_k$ in (4) are both zero matrices and we get:

$$\tilde{\mathbf{P}}_\mathbf{k} = \mathbf{P_{k-1}} \, , \tag{4a}$$

The **measurement model** (2) is substantially nonlinear. The observation vector for k-th landmark is the pair of polar coordinates, with the azimuth d_k measured relative to the agent neck direction and the range r_k measured from the agent to the landmark (Fig. 2a):

$$\mathbf{z}_k = [r_k \quad d_k]^T \ . \tag{9}$$

From the geometry of the relationship between $\mathbf{z}$ and $\mathbf{x}$ we get:

$$\mathbf{z_k} = \mathbf{h_k}(\mathbf{x}, \mathbf{v_k}) \, , \tag{2a}$$

where
$$r_k = h_k^{(1)}(\mathbf{x}, \mathbf{v_k}) = \sqrt{(x - x_{Fk})^2 + (y - y_{Fk})^2} + v_{rk} \, ,$$

$$d_k = h_k^{(2)}(\mathbf{x}, \mathbf{v_k}) = \tan^{-1}\left(\frac{y - y_{Fk}}{x - x_{Fk}}\right) - \beta + v_{dk} \, ,$$

x_{Fk}, y_{Fk} are the Cartesian coordinates of the k-th reference point $\mathbf{F}_k$, and
$\mathbf{v_k} = [v_{rk} \quad v_{dk}]^T$ is the random measurement error.

The low index k in (2a) underscores that, in general, for different landmarks the measurement model could be different. For all visible flags the measurement model (2a) stays the same, with the only difference of that the flag coordinates x_{Fk}, y_{Fk} and the variance of the range measurement error v_{rk} are individual for each flag. The magnitude of the latter error is proportional to the range r to the flag. The variance of the direction measurements, however, for all flags is same. Therefore, the covariance matrix for flags is,

$$\mathbf{R}_k = \begin{bmatrix} r_k^2 \sigma_r^2 & 0 \\ 0 & \sigma_d^2 \end{bmatrix}, \tag{10}$$

where $\sigma_d^2 = 1/(12 \cdot (180^o / \pi)^2)$ and $\sigma_r^2 = (0.01)^2 / 12$ for flags and $\sigma_R^2 = (0.1)^2 / 12$ for moving objects.

For the field borderlines the measurement model differs in that the range measurements are unavailable and only the direction can be measured. To keep the same model for both types of landmark, in the covariance matrix (10) for borderlines we assume that $\sigma_r^2 = +\infty$.

For calculating the **matrix weight** (5) we need two Jacobian matrices, $\mathbf{H}_k$, and $\mathbf{V}_k$. They immediately follow from (2a) and (10)

$$\mathbf{H}_k = \begin{bmatrix} \frac{x}{\sqrt{(x-x_{Fk})^2+(y-y_{Fk})^2}} & \frac{y}{\sqrt{(x-x_{Fk})^2+(y-y_{Fk})^2}} & 0 \\ -\frac{y-y_{Fk}}{(x-x_{Fk})^2+(y-y_{Fk})^2} & \frac{x-x_{Fk}}{(x-x_{Fk})^2+(y-y_{Fk})^2} & -1 \end{bmatrix}, \qquad (11)$$

$$\mathbf{V}_k = \mathbf{V} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}. \qquad (12)$$

Therefore, the matrix weight can be calculated, as follows:

$$\mathbf{K_k} = \mathbf{P_{k-1}H_k^T(H_kP_{k-1}H_k^T + R_k)^{-1}}. \qquad (5a)$$

This weight is used to compute the optimal state variable vector estimate

$$\mathbf{\hat{x}_k} = \mathbf{\hat{x}_{k-1}} + \mathbf{K_k}(\mathbf{\hat{z}_k} - \mathbf{h}_k(\mathbf{\hat{x}_{k-1}}, \mathbf{0})), \qquad (6a)$$

and its covariance matrix

$$\mathbf{P_k} = \mathbf{(I - K_kH_k)P_{k-1}}. \qquad (7a)$$

Expressions (3a), (4a), (5a), (6a), (7a), coupled with (10), (11), (12) yield the sought optimal algorithm for agent localization in the simulated soccer. For total *N* visible landmarks it must be executed *N* times for k=1,2,…,*N*.

The **initial state** $(\mathbf{\hat{x}_0}, \mathbf{P_0})$ for this algorithm is being set depending on the number of available measurements. Fig. 4 gives the idea of how many flags can be viewed by the soccer agent with 2000 random, uniformly distributed locations and face directions.

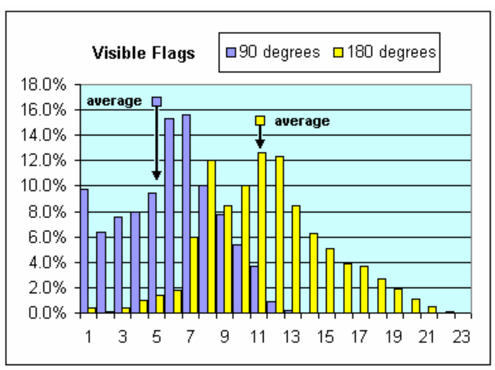

Fig.4. Number of visible flags for different viewing sector widths

In all cases the *a priori* estimated location of the agent extrapolated from the previous simulation cycle is standing for $\mathbf{\hat{x}}_0$. The choice of the covariance matrix $\mathbf{P}_0$ depends on the number of visible flags.

For the 90-dergee viewing sector width, in about 91 per cent of cases, two or more flags are visible to the agent, and at least one borderline is always visible. In this situation the error of the *a priori* estimated location of the agent $\mathbf{\hat{x}}_0$ extrapolated from the previous simulation cycle is at least 2..3 times greater than the error resulting from using for self-localization just one borderline and two flags. With more flags, this difference only increases. This allows making the following assumption about the initial state of the Kalman filter:

$$\mathbf{P}_0 = \begin{bmatrix} +\infty & 0 & 0 \\ 0 & +\infty & 0 \\ 0 & 0 & +\infty \end{bmatrix}. \qquad (13)$$

In fact, this means that the previously estimated agent location must be ignored, because (5a) assigns very large weight to the innovation component in (6a).

In the rare cases when less than two flags are visible, ignoring history data would result in the increased errors. Therefore, the diagonal elements in $\mathbf{P}_0$ are set to finite positive values.

4.3 Discussion

Recalling that the Kalman filter algorithm yields the maximum-likelihood estimate for Gaussian statistics only, it is reasonable to analyze how close this assumption is to the real situation.

In fact, by assuming Gaussian statistics, we approximated the ambiguity area BCDE in Fig.1a by an ellipsis and made the preference of its center over the rest points (Fig.5) In doing so, we are still preserving the means and standard deviations of the random errors measured in the longitudal and latitudal directions and their correlation.

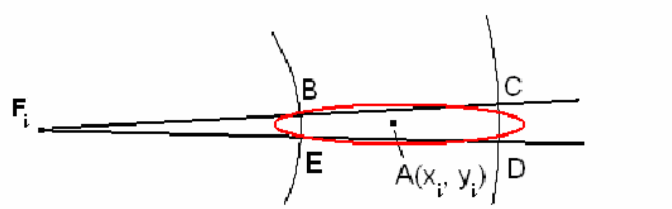

Fig.5. Approximating the ambiguity area with an ellipsis

Therefore, it is safe to say that the precision of the estimate derived above would be very close to what could be potentially obtained in the ideal case, when all the peculiarities of the measurement error statistics had been taken into account. In Section 5 below we demonstrate that the loss in precision can be indeed neglected.

The Kalman filter theory assumes that the measurement noise is uncorrelated between observations. From the Soccer Server design it follows that this assumption always holds with only one exception. There are pairs of parallel borderlines. For each pair, angular measurement errors have identical values. This is violating the assumption that errors in (2) are the discrete white noise. We walk around this artifact by using only the nearest line of two visible parallel ones.

We have neglected the simulated soccer agent movement over time for several reasons. Precise modeling agent dynamics is a non-trivial problem because the agent movements are rather complex. With a simpler state transition function (e.g. linear extrapolation), the random component would not be a white sequence because it is strongly correlated between simulation cycles. This would require more sophisticated algorithm than we have just considered. On the other hand, it follows from our simulation results below that the self-localization error in each taken separately simulation cycle can be made several times less than the error resulting from the disturbances in the regular agent movement. In other words, matrix $\mathbf{Q}_k$ would be dominating in (4). This would be inhibiting the contribution of previous measurements in (6) making the innovation contribution the main contributor to the final estimate $\hat{\mathbf{x}}_k$. The bottom line is that the data from previous cycles would not be contributing much to the overall precision

Joint estimation of the agent Cartesian coordinates with its neck direction angle is automatically taking into account the correlation of Cartesian coordinate measurements, which was a point of concern for some of our predecessors who just neglected it for the sake of simplicity. In another study [10] we showed that neglecting correlation in self localization increases the error by 10%. Kalman filtering also is leaving no chance for the angular systematic error, which emerges in the algorithms proposed by far. This allows us to claim that the algorithm proposed here is utilizing practically all the available information.

5 Simulation Results

To determine the precision of the proposed method, we have run five sets of simulations. Each of them contained total 2000 experiments by placing the agent in the random points which were uniformly spread over the field. The neck direction was uniformly distributed in the [0°, 360°) interval. For most experiments the agent viewing sector was 90 degrees wide. Some experiments have been run at 180 degrees. The obtained results are accurate with less than ±0.9% error, 19 times of 20. They are summarized in Table 2.

Table 2. Experiments with the self-localization algorithms (90-dergee agent viewing sector)

Experiment ID	Description	Objective	Localization error, meters
1	Kalman Filter as proposed in Section 4	Estimating absolute accuracy	0.091 (100%)
2	Kalman Filter with Gaussian measurement error statistics	Estimating loss of precision from non-Gaussian statistics	0.090 (98.9%)
3	Kalman Filter with ignored angular measurements	Estimating the gain from using angular data	0.13 (143%)
4	All-flags, with systematic angular error present	Comparison with the widely used algorithm	0.11 (121%)
5	Nearest flag algorithm	Comparison with the simplest method	0.30 (330%)

Experiment #1 estimates **the achievable accuracy of the proposed algorithm** and provides a yardstick for comparing it with the alternative self-localization methods.

Experiment #2 estimated **the loss in precision which resulted from violating the assumption that the visual sensor measurement errors are Gaussian**. For this purpose, we compared the self-localization errors in two cases:

1. default Soccer Server rounding errors (±0.5° for the angle and ±0.5% for the range), and
2. Gaussian errors having zero means and same standard deviations (i.e. 0.289° and 0.289%, respectively).

We found that in the Gaussian case, for which our estimator is only strictly optimal, the mean error is 98.9% of the error obtained in the default case. This gives the idea of the price paid for the sub optimality. Since the difference is very close to the accuracy limit of our experiments, it is clear that the Gaussian assumption only insignificantly affects the estimator quality.

Experiment #3 evaluated **the loss in precision when angular measurements are ignored**, as suggested by some of our predecessors. The test was run by setting in (10) the angular variance σ_d^2 1000 times greater that it originally was. This forced the Kalman filter to ignore angular measurements and resulted in a 43% increase of the error. By revisiting Table 1 one can figure out that for the model BASIC UvA team this loss would cause about 40% reduction in chance of winning a game against the similar opponent which is processing the angular data properly.

Experiment #4 **evaluates the presumably commonly used self-localization algorithm**. It is using one border line for estimating the neck direction angle and then is utilizing all visible flags to estimate the agent location. No care is taken of the systematic angular error, like shown in Fig.3. Although this algorithm has been developed by us, it is combining the ideas, which are widely spread in the simulated soccer community. We speculate that it could be somewhat more accurate that each of its predecessors, because it is taking correlation of the Cartesian coordinates measurements into account. Still compared to the Kalman filter, it results in the self-localization error which is greater by 21%. We have also run similar experiment with a 180 degree visual sector, which resulted in a 45% increase of the error. Assuming that 90- and 180-degree sectors are being used with equal probability, the average localization error would be greater by 33%. It follows from Table 1 that such a loss in the accuracy could cost reduction of chance of winning a game by more than 25% while playing against similar soccer team using the Kalman filter. It can be seen from Fig. 4 how many flags are available for the simulated soccer agent for self-localization. With 90-dergee viewing sector the agent can see two or more two flags 91% times (5.7 flags on average). With 180-degree sector this chance increases up to 99% (11.4 on average). This explains why for the wider sector the accuracy gain is higher.

Experiment #5 gives the reference point for the achievable accuracy of the proposed method as **compared to the simplest, the nearest-flag self-localization algorithm**. The Kalman filter offers a three-fold accuracy gain in this case. This is consistent with the reported gain from using the particle filter [2].

It is safe to say that Table 2 contains the minimal achievable average self-localization errors in the simulated soccer and could be used as the reference point.

In a separate set of the experiments we assessed **the benefits of two-agent ball tracking**. Two agents were simultaneously viewing the ball from different locations (Fig.1b). Each agent knew its location and orientation precisely, and could relay his estimate of the ball coordinates to the second agent without increasing the estimation error. The latter agent was using two sets of ball (x,y) coordinate measurements to estimate the ball location more precisely.

We compared two algorithms, heuristic and the Kalman filter. The heuristic algorithm was calculating the ball Cartesian coordinates based on the range and direction measurements by the agents. Then two *x,y* pairs of the ball coordinates were being merged into the final estimate using the near-optimal method based on the maximal-likelihood criteria [10] and its two modifications. One modification ignored the correlation between the Cartesian coordinates, and the second ignored the angular data.

The Kalman filter was using the joint estimate of the ball location based on the observations of its polar coordinates by the two agents.

Agents were placed 2000 times 10 meters apart in random locations at the distance from the ball uniformly distributed in the 5 to 15 meter interval. The simulation results are summarized in Table 3. The obtained results are accurate with less than ±1.1% error, 19 times of 20.

The assistance by the second agent while estimating the ball location in a 90-degree sector can potentially reduce the mean linear error about two times. Ignoring correlation would reduce this gain to just 1.3 times. Ignoring angular measurements would be highly counterproductive. However, the difference between the Kalman filter and the 'near-optimal' algorithm one is statistically insignificant. That is because of the latter algorithm by design is just another implementation of the maximal-likelihood estimator, i.e. it is equivalent to the Kalman filter.

Table 3. One- and two-agent ball tracking average linear error (in meters) using different estimation algorithms (90-degree sector)

One agent	Two agents			
	Distances-only	**Ignore-correlation**	**Near-optimal**	**Kalman Filter**
1.17 (193%)	4.10 (676%)	0. 78 (129%)	0.606 (100%)	0.601 (99.2%)

In other experiments reported elsewhere [10] we have also found that the joint estimation of the Cartesian coordinates and the agent orientation would offer more significant gains in the robotics application, where the systematic error is greater than in the simulated soccer (0.5º). In particular, we have found that that average localization error can be decreased 4..8 times, if the angular bias is uniformly distributed in the interval as big as ±4.0º. This implies that the significance of this work is not limited to the simulated soccer. Rather, with only minor changes the proposed algorithm can be reused in other robotics applications, where joint estimation of the robot location and its orientation could result in significant gains.

6 Conclusion

We have derived algorithms for determining absolute Cartesian coordinates of objects using imprecise readings of local polar coordinates, supplied by a visual sensor in a system like robotic soccer. The proposed solution is based on the Kalman filtering technique.

The innovation is in the rigorous treatment of this problem from the positions of the stochastic estimation theory. In particular, by jointly estimating the agent location and orientation, we have taken into account the correlation of the raw measurements which is emerging after converting them into Cartesian coordinate system. By far, this correlation has been neglected in the simulated robotics soccer applications, which resulted in some losses. Although in self-localization this loss is negligible, in the related task of ball tracking using agent communication negligence of correlation increases the linear error by 30 per cent.

Using the joint estimation of the agent location and orientation allows walking around the problem of the systematic error present in the angular measurements. This problem has been persisting in the self-localization algorithms published so far and limited their accuracy. In particular, we have shown that ignoring the angular data as the way to getting rid of the systematic error would result in about 43 per cent increase in the localization error.

We have demonstrated that the non-Gaussian statistics of the raw measurement errors, which presumably was a concern for some researcher who have been reluctant about using Kalman filter in the simulated soccer on these grounds, is indeed not an obstacle. Our experiments has shown that replacing the non-Gaussian errors with the equivalent 'true' normal discrete noise would not result in statistically significant differences in the agent self-localization accuracy. Compared to the best published algorithms, the new method can reduce the average error by 25-33 per cent.

Because we have shown that the assumption made about Gaussian statistics does not reduce the precision, we can guarantee that, the solution found cannot be tangibly improved in terms of the mean error of the location estimate. We thus indeed have utilized all the potential of the visual sensor, as it applies to single simulation cycle.

The computational effort required for implementing this algorithm is proportional to the number of the landmarks used for self-localization. Compared with the nearest flag method, the increase in the computation time is roughly six-fold. We believe that it is a fair cost for gaining a three times higher accuracy than the nearest-flag algorithm.

We also have found that the proposed algorithm could potentially offer even more gains in cases when the systematic angular errors are greater that those present in the simulated soccer. Therefore we hope that after some modifications, our algorithm could be reused in some other robotics applications.

The results, however, are limited to data processing in a single simulation cycle. Our future work will be targeted at similar comprehensive study of the coordinate data processing over time using all the power of methods offered by the Kalman filtering technique.

References

1. Bach, J., Gollin, M.: Self-Localization revisited. In: Birk, A., Coradeschi, S., Tadokoro, S. (eds.): RoboCup 2001, Lecture Notes in Artificial Intelligence, Vol. 2377. Springer-Verlag Berlin Heidelberg New York (2002) 251-256
2. de Boer, R., Kok, J.: The Incremental Development of a Synthetic Multi-Agent System: The UvA Trilearn 2001 Robotic Soccer Simulation Team. M.S. Thesis. Faculty of Science, University of Amsterdam (2002)
3. Foroughi. E., Heintz, F., Kapetanakis, K., Kotiadis, K., Kummeneje, J., Noda, I., Obst, O., Riley, P., and Steffens, T.: RoboCup Soccer Server User Manual. Versions 7.06 and later (2001) http://sourceforge.net/projects/sserver
4. Haker, M., Meyer, A., Polani, D., and Martinez, T.: A Method for Incorporation of New Evidence to Improve World State Estimation. In: Birk, A., Coradeschi, S., Tadokoro, S. (eds.): RoboCup 2001, Lecture Notes in Artificial Intelligence, Vol. 2377. Springer-Verlag Berlin Heidelberg New York (2002) 362-367
5. Kok, J., de Boer, R., Groen, F., Vlassis, N.: UvA Trilearn 2001 - Soccer Simulation Team. Faculty of Science, University of Amsterdam (2001) http://carol.wins.uva.nl/~jellekok/robocup/2001/index_en.html
6. Müller-Gugenberger P., Wendler, J.: AT Humboldt 98 -- Design, Implentierung und Evaluierung eines Multiagentensystems für den RoboCup-98 mittels einer BDI-Architektur. Diploma thesis. Huboldt University of Berlin (1998) http://www.informatik.hu-berlin.de/~wendler/paper/gugenbergerWendler:diplom:1998.ps.gz
7. Catlin, D. (1989) Estimation, Control, and the Discrete Kalman Filter (Applied Mathematical Sciences, Vol 71) Springer Verlag, ISBN: 038796777X
8. MathWorld: Maximal Likelihood, http://mathworld.wolfram.com/MaximumLikelihood.html
9. MathWorld: Jacobian, http://mathworld.wolfram.com/Jacobian.html
10. Kyrylov, V., Brokenshire, D., Hou, E. Optimizing Precision of Self-Localization in Simulated Soccer Agents. Technical report SIAT-TR-VK-01-2003.